Partial Differential Equations Differential Equations X. Du

- Referred to as PDEs
- These are differential equations involving partial derivatives
- Examples:
    - Heat equation
        - One dimension: $u_t = \alpha u_{xx}$.
            - $u(t,x)$ represents temperature with respect to time, position
            - α is the rate of diffusion.
        - Generalized: $\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$
    - Wave equation
        - $\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0$
    - Schrödinger equation
        - Time-dependent: $i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t) = \hat{H} \Psi(\vec{r},t)$
    - Navier-Stokes equations
- Existence and uniqueness are hard to prove
- **Superposition Principle** (pretty much the same as for ODEs): any linear combination of solutions of a homogeneous linear PDE is also a solution.
- **Second order linear partial differential equations**:
    - Take the form $Au_{xx} + 2Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu + G = 0$:
    - **Elliptic PDE**: $B^2 - AC < 0$
        - Examples: Laplace equation $\nabla^2 u = 0$, Poisson equation $\nabla^2 u = f(x,y)$
    - **Hyperbolic PDE**: $B^2 - AC > 0$
        - Example: wave equation in one dimension $u_{tt} = c^2 u_{xx}$
    - **Parabolic PDE**: $B^2 - AC = 0$
        - Example: heat equation in one dimension $u_t = \alpha u_{xx}$